

Title: Systems of Linear Equations – Substitution Method

Class: Math 100 or Math 107

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Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.

Keywords/Tags: systems, systems of linear equations, substitution, consistent

Systems of Linear Equations – Substitution Method

Purpose:

This is intended to refresh your knowledge about solving systems of linear equations using the substitution method, where there is a single solution.

Recall that a **system of equations** consists of two or more equations each with two or more variables. A **solution** to a system in two variables is an ordered pair (x, y) that satisfies each equation in the system. For now, we will concentrate on systems of linear equations.

Substitution Method – Solve for one of the variables in one of the equations and substitute it into the other equation. After this is done, you will have a single equation with one variable – solve for it. Then back-substitute to find the other.

Example: Solve $\begin{cases} x - 3y = -2 \\ 5x + 3y = 17 \end{cases}$ using the substitution method.

Let's solve the first equation for x , since this requires the least work: $x = 3y - 2$

We now substitute this for x in the second equation: $5x + 3y = 17$
 $\Rightarrow 5(3y - 2) + 3y = 17$

Notice that we now have a single equation with the variable y . Let's solve for y :

$$\begin{aligned} 5(3y - 2) + 3y &= 17 \\ \Rightarrow 15y - 10 + 3y &= 17 \\ \Rightarrow 18y - 10 &= 17 \\ \Rightarrow 18y &= 27 \\ \Rightarrow y &= \frac{27}{18} \\ \Rightarrow y &= \frac{3}{2} \end{aligned}$$

Now we can go back to our first step and solve for x :

$$\begin{aligned} x &= 3y - 2 \\ \Rightarrow x &= 3\left(\frac{3}{2}\right) - 2 \\ \Rightarrow x &= \frac{9}{2} - 2 \\ \Rightarrow x &= \frac{5}{2} \end{aligned}$$

So our solution is the ordered pair $\left(\frac{5}{2}, \frac{3}{2}\right)$. (Note that this is where the two lines intersect.)

Example: Now it's your turn. Solve $\begin{cases} x + y = 2 \\ 2x - 3y = 9 \end{cases}$ using the substitution method.

Solve the first equation for y : $y = \text{-----}$

Now substitute this for y in the second equation: -----

Notice that you now have a single equation with the variable x – solve for x :

$x = \text{-----}$
(Did you get $x = 3$? If not, go back and check your work.)

Now go back to your first step and solve for y :

$y = \text{-----}$

Did you get $(3, -1)$ for your solution? Good! Now try the next to on your own.

Example: Solve using the substitution method.

(a) $\begin{cases} 4x - y = 7 \\ -2x + 3y = 9 \end{cases}$

(b) $\begin{cases} 6x - 2y = -3 \\ 5x + 3y = 4 \end{cases}$

(The answers are $(3, 5)$ for (a) and $(-\frac{1}{28}, \frac{39}{28})$ for (b). If you did not get these, consult a tutor for help.)